Key

Review for Quiz on 4.1 through 4.3

Give the indefinite integral.

1. $\int \frac{x^2-1}{\sqrt{x}}\,dx$

$\int (x^{3/2} - x^{-1/2})dx$

$\frac{2}{5}x^{5/2} - 2x^{1/2} + C$

Answer: $\frac{2}{5}x^{5/2} - 2x^{1/2} + C$

2. $\int 3\sin x - 2\sec^2 x\,dx$

$-3\cos x - 2\tan x + C$

Answer: $-3\cos x - 2\tan x + C$

3. $\int \tan x \cos x\,dx$

$\int \frac{\sin x}{\cos x} \cdot \cos x\,dx$

$-\cos x + C$

Answer: $-\cos x + C$

4. a. Give the general solution for the differential equation $\frac{dy}{dx} = 2x - 1$.

$y = \int (2x-1)\,dx$

$x^2 - x + C$

Answer: $y = x^2 - x + C$

b. Give the particular solution, which contains the point (1, 3).

$3 = (1)^2 - (1) + C$

$3 = 1 - 1 + C$

$C = 3$

Answer: $y = x^2 - x + 3$

5. Find f(x) if $f''(x) = \sin x$, $f'(0) = 1$, and $f(0) = 6$. $f(x) = -\sin x + 2x + 6$

$f'(x) = \int \sin x\, dx$

$f'(x) = -\cos x + C$

$1 = -\cos 0 + C$

$1 = -1 + C$

$C = 2$

$f'(x) = -\cos x + 2$

$f(x) = \int (-\cos x + 2)\, dx$

$f(x) = -\sin x + 2x + C$

$6 = -\sin(0) + 2(0) + C$

$C = 6$

6. On the moon the acceleration due to gravity is -1.6 meters per second per second. A stone is dropped from a cliff on the moon and hits the surface of the moon 20 seconds later.

a. How far did it fall? 320 meters

$s''(t) = -1.6$

$s'(t) = v(t) = \int -1.6\, dt = -1.6t + V_0 = -1.6t$

$s(t) = \int -1.6t\, dt = -.8t^2 + s_0$

height of cliff

$0 = -.8(20)^2 + s_0$ $\qquad s_0 = .8(20)^2 = 320$ meters

b. What was its velocity at impact? -32 meters/sec

$v(t) = -1.6t$

$v(20) = -32$

7. Find the sum. $\sum_{i=1}^{6}(3i+2)$ 75

$3\sum_{i=1}^{6} i + \sum_{i=1}^{6} 2 = 3 \cdot \frac{6(7)}{2} + 2(6) =$

$3\frac{n(n+1)}{2}$ $\qquad 2(6)$

8. Use sigma notation to write the sum.

$$\frac{1}{5(1)} + \frac{1}{5(2)} + \frac{1}{5(3)} + \ldots + \frac{1}{5(11)}$$

$\sum_{i=1}^{11} \frac{1}{5i}$

Summation Formulas

$\sum_{i=1}^{n} c = cn$ $\qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

9. Using the above Summation Formulas evaluate the sum. $\sum_{i=1}^{20}(i-1)^2$

$$\sum_{i=1}^{20} i^2-2i+1 = \sum i^2 - 2\sum i + \sum 1$$

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$$\frac{n(n+1)(2n+1)}{6} - \frac{\cancel{2}(n)(n+1)}{\cancel{2}} + n$$

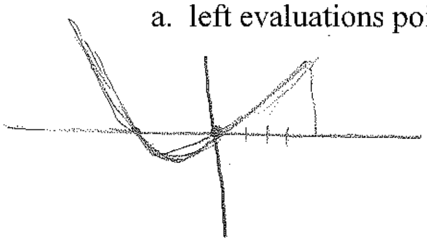
$n=20$

$$\frac{20(21)(41)}{6} - 20(21) + 20$$

10. Find an approximation of the area bounded by f(x) = x(x + 2) and the x-axis on the interval [0, 2] using 4 subintervals and

$$\sum_{i=0}^{3}\left[\tfrac{i}{2}\left(\tfrac{i}{2}+2\right)\right]\left(\tfrac{1}{2}\right) = 4.75$$

a. left evaluations points



x_i	$f(x_i)$
0	0
.5	1.25
1	3
1.5	5.25

Area $= \frac{1}{2}\sum f(x_i) = \frac{1}{2}(9.5)$

b. midpoints

6.625

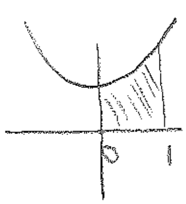
width is still $\frac{1}{2}$

x_i	$f(x_i)$
.25	.5625
.75	2.0625
1.25	4.0625
1.75	6.5625

Area $= \frac{1}{2}\sum f(x_i) = \frac{1}{2}\cdot 13.25$

11. Use the limit process to find the area of the region between the graph of the function and the x-axis over given interval. Sketch the region.

y = x^2 + 2, [0, 1]



$2\frac{1}{3}$

$$\sum_{i=1}^{\infty} f\left(\tfrac{i}{n}\right)\cdot\tfrac{1}{n} = \sum_{i=1}^{\infty}\left[\left(\tfrac{i}{n}\right)^2+2\right]\left(\tfrac{1}{n}\right)$$

$$\frac{1}{n}\sum\frac{i^2}{n^2} + \frac{1}{n}\sum 2$$

$$\frac{1}{n^3}\sum i^2 + \frac{1}{n}\sum 2$$

$$\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{n}\cdot 2n$$

$$= \frac{n(2n^2+3n+1)}{6n^3} + 2 = \frac{2n^3+3n^2+n}{6n^3} + 2$$

$$\lim_{n\to\infty}\left[\frac{1}{3} + \cancel{\frac{1}{2n}}^{0} + \cancel{\frac{1}{6n^2}}^{0} + 2\right] = 2\frac{1}{3}$$

12. Given $\int_1^5 f(x)dx = \frac{7}{3}$, find:

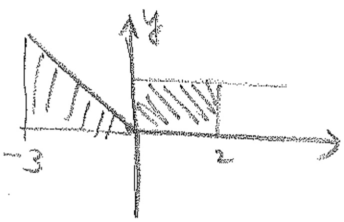
a. $\int_1^1 f(x)dx = 0$ $\underline{\quad 0 \quad}$

b. $\int_5^1 f(x)dx = -\frac{7}{3}$ $\underline{\quad -7/3 \quad}$

c. $\int_1^3 f(x)dx + \int_3^5 f(x)dx = \frac{7}{3}$ $\underline{\quad 7/3 \quad}$

13. Let $f(x) = \begin{cases} 1, & x \geq 0 \\ -x, & x < 0 \end{cases}$. Evaluate each integral.

a. $\int_{-3}^{2} f(x)dx$ $\underline{\quad 13/2 \quad}$



$\int_{-3}^{0} f(x)dx = \frac{1}{2}(3)(3) = \frac{9}{2}$

$\frac{9}{2} + 2$

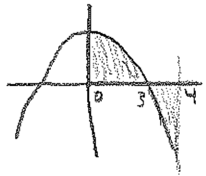
$\int_0^2 f(x)dx = (2)(1) = 2$

b. $\int_{-k}^{k} f(x)dx = -\frac{1}{2}(k)(k) + (k)(1)$ $\underline{\quad -\frac{1}{2}k^2 + k \quad}$

Clarity

14. Write the definite integral(s) that represents the area of the region bounded by the graphs of $y = 9 - x^2$, $x = 0$, $x = 4$, and $y = 0$.

$y = -x^2 + 9 = -(x-3)(x+3)$



$\boxed{\int_0^3 (9-x^2)dx - \int_3^4 (9-x)^2 dx}$