Name: Key

Period: ____

Calculus: Chapter 4 Review Sheet

Summation Formulas

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. Using the summation formulas above, evaluate $\sum_{i=1}^{20} i(1+3i)^2$.

$\underline{414,330}$

$$\sum_{i=1}^{20} i(1+6i+9i^2) = \sum i + 6\sum i^2 + 9\sum i^3$$

$$\frac{20(21)}{6} + \frac{6(20)(21)(41)}{6} + \frac{9(20)^2(21)^2}{4}$$

$$210 + 17,220 + 396,900$$

2. Use summation formulas and the limit process, find the net area of the region between the graph of the function and the x-axis over the given interval.
Sketch the region. $y = 1 - x^2$ $\quad [0,3]$

$\underline{-6}$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$\sum_{i=1}^{n} f\left(\frac{3}{n}i\right)\cdot\frac{3}{n} = \sum\left[1-\left(\frac{3}{n}i\right)^2\right]\left(\frac{3}{n}\right) = \frac{3}{n}\sum\left(1-\frac{9i^2}{n^2}\right) = \frac{3}{n}\sum 1 - \frac{27}{n^3}\sum i^2$$

$$\frac{3}{n}\cdot n - \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} = 3 - \frac{27(2n^3+3n^2+n)}{6n^3} = 3 - 9 + \frac{27}{2n} + \frac{9}{2n^2}$$

$$\lim_{n\to\infty}\left(-6 + \cancel{\frac{27}{2n}}^{0} + \cancel{\frac{9}{2n^2}}^{0}\right) = -6$$

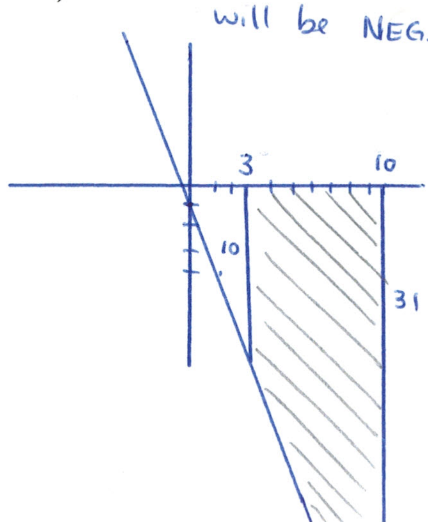
Question
why is this
a negative answer?

3. For the following:
a. Graph and shade the region represented by the definite integral.
b. Evaluate the definite integral using geometry.

$$\int_3^{10} (-3x-1)\,dx$$

area below x-axis
will be NEGATIVE

a.

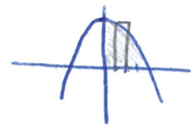


b. $\underline{-143.5}$

$$A_{trap} = \tfrac{1}{2}(10+31)(7)$$

4. Use a Riemann Sum to approximate the area bounded by $f(x) = 4 - x^2$ and the x-axis on the interval [0, 2] using 4 subintervals and

a. left endpoints $\Delta x = \frac{2-0}{4} = \frac{1}{2}$



$A \approx$ 6.25 circumscribed rectangles

x_i	$f(x_i)$
0	4
$\frac{1}{2}$	3.75
1	3
$\frac{3}{2}$	1.75

$\text{Area} = \sum f(x_i)\cdot\Delta x = \Delta x \sum f(x_i) = \frac{1}{2}\cdot 12.5$

$\sum f(x_i) = 12.5$

b. right endpoints

$A \approx$ 4.25 inscribed rectangles

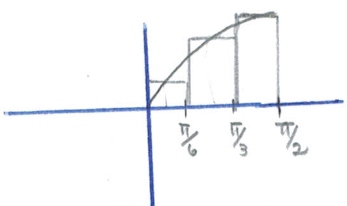
x_i	$f(x_i)$
$\frac{1}{2}$	3.75
1	3
$\frac{3}{2}$	1.75
2	0

$\Delta x = \frac{1}{2}$

$\sum f(x_i) = 8.5$

$\text{Area} = \Delta x \sum f(x_i) = \frac{1}{2}(8.5)$

5. Use a Riemann Sum to approximate the area bounded by $f(x) = \sin x$ and the x-axis on the interval $[0, \frac{\pi}{2}]$ using 3 subintervals and midpoints.



$\Delta x = \frac{\pi/2 - 0}{3} = \frac{\pi}{6}$

interval	mid pt	height @ midpt	height
$0 - \pi/6$	$\pi/12$	$\sin(\pi/12)$	.2588
$\pi/6 - \pi/3$	$\pi/4$	$\sin(\pi/4)$	.7071
$\pi/3 - \pi/2$	$5\pi/12$	$\sin(5\pi/12)$	.9659

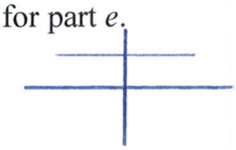
$A \approx$ 1.012

$\frac{\pi}{6}(.2588 + .7071 + .9659)$

$\frac{\pi}{6}(1.9318) \approx 1.012$

6. Consider the symmetry of odd and even functions in parts *a* through *d*. If the statement is true, circle TRUE. If the statement is false, circle FALSE and then correct the statement to make it true. Fill in the blank for part *e*.

a. TRUE or FALSE. $\int_{-a}^{a} 3dx = 2\int_{-a}^{0} 3dx$



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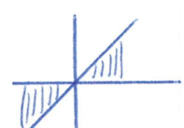
b. TRUE or FALSE. $\int_{-a}^{a} x\,dx = 0$



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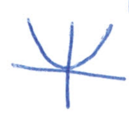
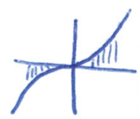
c. TRUE or FALSE. $\int_{-a}^{a} x^2 dx = 2\int_{0}^{a} x^2 dx$



F $= 2\int_0^a x^2 dx$

d. TRUE or FALSE. $\int_{-a}^{a} x^3 dx = \cancel{2\int_{0}^{a} x^3 dx} = 0$



F $= 0$

e. Fill in the blank. $\int_{-2}^{6} f(x)dx - \int_{-2}^{2} f(x)dx = \int_2^6 f(x)dx$

7. Find the average value of the function over the given interval. Then find c that is guaranteed by the Mean Value Theorem for Integrals.

f(c) = 3

$f(x) = \dfrac{9}{x^2}$ [1, 3]

c = $\sqrt{3}$

$$f(c) = \frac{1}{2}\int_1^3 \frac{9}{x^2}\,dx = \int_1^3 \frac{9}{2}x^{-2}\,dx = -\frac{9}{2}x^{-1}\Big|_1^3 = -\frac{9}{2}\left(\frac{1}{3}-1\right) = -\frac{9}{2}\cdot -\frac{2}{3} = 3$$

$\frac{9}{x^2} = 3$ $\quad x^2 = 3$

$x = \pm\sqrt{3}$ only $+\sqrt{3}$ is in the interval $[1,3]$

8. If f″(x) = 4 and f′(2) = 3 and the graph goes through the point (1, 6), find f(x).

$f'(x) = \int f''(x)\,dx = \int 4\,dx = 4x + C$

$f(x) = 2x^2 - 5x + 9$

$f'(2) = 4(2) + C = 3$

$C = -5$

$f(x) = \int (4x-5)\,dx$

$= 2x^2 - 5x + C$

$f(1) = 2(1)^2 - 5(1) + C = 6$

$= 2 - 5 + C = 6$

$-3 + C = 6$

$C = 9$

w/o technology

9. Evaluate the following integrals:

a. $\int \frac{4}{x^3}dx = \int 4x^{-3}\,dx = -\frac{4}{2}x^{-2} + C$ $\qquad -2x^{-2} + C$

b. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\,dx = \tan x\Big|_{\pi/6}^{\pi/3}$ $\qquad 2\sqrt{3}/3$

$= \tan(\pi/3) - \tan(\pi/6) = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

c. $\int_1^4 \frac{x^3 - x}{x}dx = \int_1^4 (x^2 - 1)\,dx$ $\qquad 18$

$\frac{1}{3}x^3 - x\Big|_1^4 = \left(\frac{1}{3}\cdot 4^3 - 4\right) - \left(\frac{1}{3}\cdot 1^3 - 1\right) = 17.\bar{3} - (-.\bar{6})$

d. $\int \frac{x}{3\sqrt{x^2 - 8}}dx$ $\qquad u = x^2 - 8$ $\qquad du = 2x\,dx$ $\qquad \frac{1}{3}(x^2-8)^{\frac{1}{2}} + C$

$\frac{1}{6}\int \frac{du}{\sqrt{u}} = \frac{1}{6}\int u^{-1/2}\,du = \frac{1}{3}u^{1/2} + C$

e. $\int \tan^4 x \sec^2 x\,dx = \int u^4\,du$ $\qquad \frac{1}{5}\tan^5 x + C$

$u = \tan x$ $\qquad = \frac{1}{5}u^5 + C$

$du = \sec^2 x\,dx$

$u = 8 - x^{3/2}$

$du = -\frac{3}{2} x^{1/2} dx$

f. $\int \pi\sqrt{x}\left(8 - x^{\frac{3}{2}}\right)dx$ $\quad$ $-\frac{\pi}{3}\left(8 - x^{3/2}\right)^2 + C$ or

$-\frac{2\pi}{3}\int u\,du = -\frac{2\pi}{3}\cdot\frac{1}{2}u^2 + C$ or $8\pi\int x^{\frac{1}{2}}dx - \pi\int x^2 dx = \boxed{\frac{16\pi}{3}x^{3/2} - \frac{\pi}{3}x^3 + C}$

g. $\int \frac{2\cos x}{\sin^3 x}dx$ $\quad$ $-\csc^2 x + C$ OR

$u = \sin x$

$du = \cos x\,dx$

$\int 2u^{-3}du = 2\cdot\frac{-1}{2}u^{-2} + C = -u^{-2} + C = -(\sin x)^{-2} + C = \boxed{\frac{-1}{\sin^2 x} + C}$

h. $\int \frac{4}{(2-3x)^2}dx$ $\quad$ $\frac{4}{3}(2-3x)^{-1} + C$

$u = 2 - 3x$

$du = -3dx$

$-\frac{4}{3}\int u^{-2}du = \frac{4}{3}u^{-1} + C$

j. $\int t^2\sqrt{t^3 - 4}\,dt = \frac{1}{3}\int u^{1/2}du$ $\quad$ $\frac{2}{9}(t^3 - 4)^{3/2} + C$

$u = t^3 - 4$

$du = 3t^2 dt$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}u^{3/2} + C$

k. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}dx$ $\quad$ 1

$u = 1 + 2x^2$

$du = 4x\,dx$

$\frac{1}{4}\int_1^9 u^{-1/2}du = \frac{1}{4}\cdot 2u^{1/2} \Rightarrow \frac{1}{2}u^{1/2}\Big|_1^9 = \frac{1}{2}(\sqrt{9} - \sqrt{1}) = \frac{1}{2}(3-1) = \frac{1}{2}\cdot 2$

l. $\int_0^{\frac{\pi}{2}} \cos\left(\frac{2\theta}{3}\right)d\theta$ $\quad$ $\frac{3\sqrt{3}}{4}$

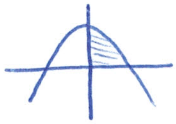
$u = \frac{2}{3}\theta$

$du = \frac{2}{3}d\theta$

$\frac{3}{2}\int_0^{\pi/3}\cos u\,du = \frac{3}{2}\sin u\Big|_0^{\pi/3} = \frac{3}{2}\left[\sin(\pi/3) - \sin(0)\right]$

$\frac{3}{2}\left[\frac{\sqrt{3}}{2} - 0\right]$

10. Find the area bounded by the positive x-axis, the positive y-axis, and the parabola $y = -x^2 + 4$. Show your work. Compare your answer to the approximations you obtained in #4.



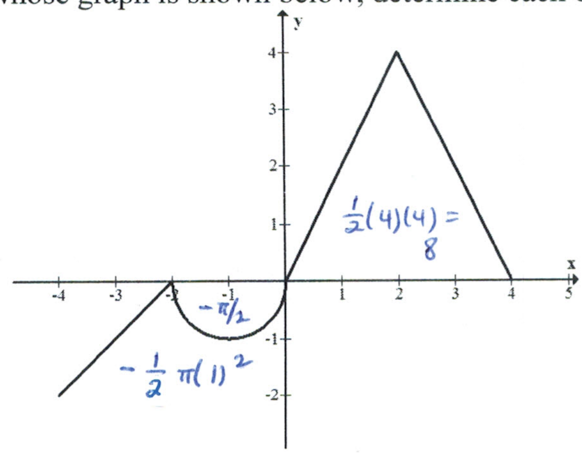
$$\int_0^2 (-x^2+4)\,dx = -\frac{1}{3}x^3 + 4x \Big|_0^2$$

$$\left[-\frac{8}{3}+8\right] - 0 = 5.\overline{3}$$

A = $5\frac{1}{3}$

inbetween the over and under estimates from #4. (Not equal to the average of the two estimates.)

11. For the function g whose graph is shown below, determine each of the following:

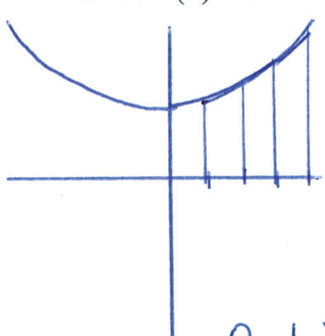


a. $\int_0^4 (g(x)+2)dx = \int_0^4 g(x)\,dx + \int_0^4 2\,dx = 8 + (2)(4)$ $\quad$ 16

b. $\int_{-2}^4 g(x)dx = -\frac{\pi}{2} + 8$ $\quad$ $8 - \pi/2$

c. $\int_4^0 g(x)dx = -\int_0^4 g(x)dx = -8$ $\quad$ -8

12. Let f(x) = x^2 + 3 for [1, 4]. Approximate the area using trapezoids where n = 3.



x_i	$f(x_i)$
1	4
2	7
3	12
4	19

$A \approx$ 30.5

$$\frac{1}{2}\left(4 + 2(7) + 2(12) + 19\right)\underbrace{(1)}_{\Delta x}$$

Question: Is this an over- or under-estimate?

13. Fill in the analogy.
Summation(Σ) : discrete function :: Integration ($\int$) : continuous function