Name: Key
Period: ____

Calculus Quiz Review: 5.1, 5.2, and 5.4
The Calculus of Logarithmic and Exponential Functions

Find y′.

1. $y = e^{x^3}$

$y' = e^{x^3} \cdot 3x^2$

$\boxed{y' = 3x^2 e^{x^3}}$

2. $y = x^2 e^{-x}$

$y' = x^2 e^{-x}(-1) + e^{-x}(2x)$

$\boxed{y' = -x^2 e^{-x} + 2xe^{-x}}$

$\boxed{y' = \frac{2x - x^2}{e^x}}$ or

3. $y = \ln(4x^2 + 3x)$

$y' = \frac{1}{4x^2+3x} \cdot 8x+3$

$\boxed{y' = \frac{8x+3}{4x^2+3x}}$

4. $y = \cos(\ln x)$

$y' = -\sin(\ln x) \cdot \frac{1}{x}$

$\boxed{y' = \frac{-\sin(\ln x)}{x}}$

5. $y = \ln(\ln\sqrt{x})$

$y' = \frac{1}{\ln\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

$\boxed{y' = \frac{1}{2x\ln\sqrt{x}}}$

(or apply a log rule first)

6. $y = \ln\left(\frac{x(x^2-2)^{\frac{2}{3}}}{\sqrt{x+4}}\right)$

$y = \ln x + \frac{2}{3}\ln(x^2-2) - \frac{1}{2}\ln(x+4)$

$y' = \frac{1}{x} + \frac{2}{3} \cdot \frac{1}{x^2-2} \cdot 2x - \frac{1}{2} \cdot \frac{1}{x+4} \cdot 1$

$\boxed{y' = \frac{1}{x} + \frac{4x}{3(x^2-2)} - \frac{1}{2(x+4)}}$

7. $y = \frac{e^{x^2+4x}}{\ln x}$

$y' = \frac{\ln x \cdot e^{x^2+4x} \cdot (2x+4) - e^{x^2+4x} \cdot \frac{1}{x}}{[\ln x]^2}$

$\boxed{y' = \frac{x\ln x\,(2x+4)e^{x^2+4x} - e^{x^2+4x}}{x[\ln x]^2}}$

8. $y = e^{x\tan x}$

$y' = e^{x\tan x} \cdot [x \cdot \sec^2 x + \tan x \cdot 1]$

$\boxed{y' = e^{x\tan x}(x\sec^2 x + \tan x)}$

9. $xe^y + 5 = xy$ Implicit Differentiation

$xe^y \cdot \frac{dy}{dx} + e^y \cdot 1 + 0 = x\frac{dy}{dx} + y \cdot 1$

$\frac{dy}{dx}(xe^y - x) = y - e^y$

$\boxed{y' = \frac{dy}{dx} = \frac{y - e^y}{x(e^y - 1)}}$

10. $y = (e^{2x} + e^{-2x})^5$

$\frac{du}{dx}$

$y' = 5(e^{2x} + e^{-2x})^4 \cdot [e^{2x} \cdot 2 + e^{-2x} \cdot (-2)]$

$\boxed{y' = 10(e^{2x} - e^{-2x})(e^{2x} + e^{-2x})^4}$

Same as:
$y' = 5(e^{2x} + e^{-2x})^4(2e^{2x} - 2e^{-2x})$

Evaluate.

11. $\int \frac{x+4}{x^2+8x}dx = \frac{1}{2}\int \frac{du}{u}$

$u = x^2+8x$

$du = 2x+8\,dx$

$\frac{1}{2}\ln|u|+C$

$\boxed{\frac{1}{2}\ln|x^2+8x|+C}$

12. $\int \cot x\,dx$

$\boxed{\ln|\sin x|+C}$

calculus fact

13. $\int \frac{e^{\ln x}}{5x}dx = \frac{1}{5}\int \frac{e^{\ln x}}{x}dx$

$e^{\ln x} = x$ * inverse functions *

$\frac{1}{5}\int \frac{x}{x}dx = \frac{1}{5}\int dx = \boxed{\frac{1}{5}x+C}$

14. $\int \frac{4}{x\ln x}dx = 4\int \frac{du}{u}$

$u = \ln x$

$du = \frac{1}{x}dx$

$4\ln|u|+C$

$\boxed{4\ln|\ln x|+C}$

15. $\int \frac{e^{\frac{2}{x}}}{x^2}dx = -\frac{1}{2}\int e^u du$

$u = 2/x = 2x^{-1}$

$du = -2x^{-2}dx$

$-\frac{1}{2}e^u+C = \boxed{-\frac{1}{2}e^{2/x}+C}$

16. $\int \sec(3x)dx = \frac{1}{3}\int \sec u\,du$

$u = 3x$

$du = 3dx$

$= \frac{1}{3}\ln|\sec u+\tan u|+C$

$= \boxed{\frac{1}{3}\ln|\sec 3x+\tan 3x|+C}$

17. $\int \frac{xe^{x^2}}{9+e^{x^2}}dx = \frac{1}{2}\int \frac{du}{u}$

$u = 9+e^{x^2}$

$du = e^{x^2}\cdot 2x\,dx$

$= \frac{1}{2}\ln|u|+C$

$= \boxed{\frac{1}{2}\ln(9+e^{x^2})+C}$

18. $\int e^{4-x}dx = -\int e^u du$

$u = 4-x$

$du = -dx$

$-e^u+C = \boxed{-e^{4-x}+C}$

19. $\int \frac{x+1}{x}dx = \int(1+\frac{1}{x})dx$

$\boxed{x+\ln|x|+C}$

20. $\int 3xe^{x^2+3}dx$

$u = x^2+3$

$du = 2x\,dx$

$\frac{3}{2}\int e^u du = \frac{3}{2}e^u+C$

$\boxed{\frac{3}{2}e^{x^2+3}+C}$

21. $\int e^{\ln x}\cdot\frac{1}{x}dx = \int x\cdot\frac{1}{x}dx$

$\int 1\,dx$

$\boxed{x+C}$

22. $\int \frac{e^{3x}+e^{8x}+e^x}{e^x}dx$

$= \int(e^{2x}+e^{7x}+1)dx$

$\boxed{\frac{1}{2}e^{2x}+\frac{1}{7}e^{7x}+x+C}$

23. Find the value of $\int_0^1 \frac{e^x}{e^x+1}dx. = \int_2^{1+e}\frac{du}{u}$

$u = e^x+1$

$du = e^x dx$

$\ln|u|\Big|_2^{1+e} =$ exact: $\boxed{\ln(1+e)-\ln 2} \approx$

approximate: $\boxed{\approx 0.62}$

24. Find the value of $\int_0^2 \frac{x^2-2}{x+1}dx. = \int_0^2\left[x-1-\frac{1}{x+1}\right]dx$ $\boxed{-\ln 3}$

$$\begin{array}{r} x-1 \\ x+1\overline{)\,x^2+0x-2} \\ \underline{x^2\;\;x\;\;\;\;\;\;} \\ -x-2 \\ \underline{-x-1} \\ -1 \end{array}$$

$\left[\frac{x^2}{2}-x-\ln|x+1|\right]_0^2 = \left[\frac{2^2}{2}-2-\ln|2+1|\right] - \left[\frac{0}{2}-0-\ln 1\right] = 2-2-\ln 3$

True or False. Circle the correct answer.

25. TRUE or FALSE: $\int_1^e \frac{1}{x}dx = 1$

$\ln|x| \Big|_1^e = \ln e - \ln 1$
$= 1 - 0 = 1$

26. TRUE or FALSE: $\int \cot x dx = -\ln|\sin x| + C$

No.

(repeat of #12)

27. TRUE or FALSE: $\ln\left(\frac{a^3}{5b^2}\right) = 3\ln a - 2\ln 5b$

$3\ln a - \ln 5(b)^2 = 3\ln a - (\ln 5 + \ln b^2)$
$= 3\ln a - \ln 5 - 2\ln b$

28. TRUE $\int e^x \sin e^x dx = -\cos e^x + C$

or check by differentiation of right side

$u = e^x$
$du = e^x dx$
$\int \sin u du = -\cos u + C = -\cos e^x + C$

29. Demonstrate that $\int \frac{x}{x+1}dx = x - \ln|x+1| + C$.

By integration:
$\int \frac{x+1-1}{x+1}dx = \int\left(1 - \frac{1}{x+1}\right)dx$
$x - \ln|x+1| + C$

- OR -

By differentiation:
$F(x) = x - \ln|x+1| + C$
$F'(x) = 1 - \frac{1}{x+1}$
$= \frac{x+1}{x+1} - \frac{1}{x+1} = \frac{x}{x+1} = f(x)$
checks!

30. Find the equation of the tangent line to the graph of $y = x\ln x^2$ at the point where $x = e$.

$f(x) = y = 2x\ln x$ by Log Rules
$y' = 2x\left(\frac{1}{x}\right) + (\ln x)(2)$
$y' = 2 + 2\ln x$
$f'(e) = 2 + 2\ln e = 4$

when $x = e$, $y = 2e \Rightarrow (e, 2e)$
slope @ $(e, 2e) = 4$

$y - 2e = 4(x - e)$ OR $y = 4x - 2e$

31. Given $y\ln x + y^2 = 0$, use implicit differentiation to find $\frac{dy}{dx}$.

$y\frac{1}{x} + \ln x \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(\ln x + 2y) = -\frac{y}{x}$

$\frac{dy}{dx} = \frac{-y}{x(\ln x + 2y)}$

$\frac{dy}{dx} = \frac{-y}{x(\ln x + 2y)}$

32. Find the area of the region bounded by the graphs of the equations. $\underline{-\frac{1}{2e^2} - \frac{1}{2}}$

$y = e^{-2x}$, y = 0, x = 0, x = 1

$$\int_0^1 e^{-2x}\,dx = -\frac{1}{2}\int_0^{-2} e^u\,du = -\frac{1}{2}e^u\Big|_0^{-2} = -\frac{1}{2}(e^{-2} - e^0)$$

$u = -2x$

$du = -2dx$

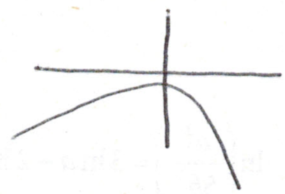
$$= -\frac{1}{2e^2} - \frac{1}{2}$$

33. Find the absolute maximum of $y = x - e^x$. y-value. $\underline{\text{maximum value}} = -1$

$f'(x) = y' = 1 - e^x = 0$

$1 = e^x$

$x = 0$



$f(0) = 0 - e^0 = -1$

$f''(x) = y'' = -e^x$

$f''(0) = -e^0 = -1 < 0$ by Second Derivative Test, local maximum occurs @ $x = 0$

34. Find the particular solution that satisfies the initial conditions. $\underline{f(x) = -\sin x + \frac{1}{4}e^{2x} + x}$

$f''(x) = \sin x + e^{2x}$, $f(0) = \frac{1}{4}$, $f'(0) = \frac{1}{2}$

$$f'(x) = \int f''(x)dx = \int \sin x + e^{2x}\,dx$$

$$f'(x) = -\cos x + \frac{1}{2}e^{2x} + C$$

$$f'(0) = -\cos 0 + \frac{1}{2}e^{2\cdot 0} + C = \frac{1}{2}$$

$$-1 + \frac{1}{2} + C = \frac{1}{2}$$

$$C = 1$$

$$f'(x) = -\cos x + \frac{1}{2}e^{2x} + 1$$

$$f(x) = \int f'(x)dx = \int\left[-\cos x + \frac{1}{2}e^{2x} + 1\right]dx$$

$$f(x) = -\sin x + \frac{1}{4}e^{2x} + x + C$$

$$f(0) = -\sin 0 + \frac{1}{4}e^0 + 0 + C = \frac{1}{4}$$

$$C = 0$$

$$f(x) = -\sin x + \frac{1}{4}e^{2x} + x$$