Name: Key

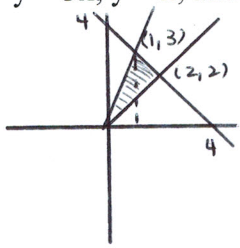
Calculus: Review for Test on 7.1 and 7.2

Reference:

$\int \tan u\,du = -\ln|\cos u| + C$ $\qquad$ $\int \cot u\,du = \ln|\sin u| + C$

$\int \sec u\,du = \ln|\sec u + \tan u| + C$ $\qquad$ $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

1. Set up the integral that would be used to find the area between the given curves.
   $y = 3x$, $y = x$, and $x + y = 4$ $\;\to y = -x + 4$



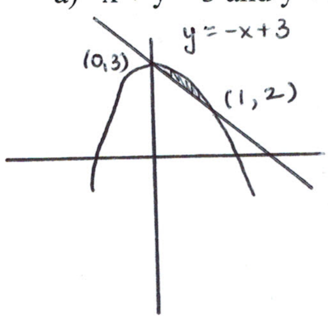
$$\int_0^1 (3x - x)\,dx + \int_1^2 ((-x+4) - x)\,dx$$

$$A = \int_0^1 2x\,dx + \int_1^2 (-2x+4)\,dx$$

2. Find the area bounded by the graphs of the given curves.

   a) $x + y = 3$ and $y + x^2 = 3$ $\qquad$ $A = 1/6$

   $y = -x + 3$ $\qquad$ $y = -x^2 + 3$



$-x + 3 = -x^2 + 3$

$-x = -x^2$

$x = x^2$

$x = 0, 1$

$$\int_0^1 [(-x^2+3) - (-x+3)]\,dx$$

$$\int_0^1 (-x^2 + x)\,dx$$

   b) $x = y^2$ and $x - y = 2$ $\;\to x = 2 + y$ $\qquad$ $A = 4.5$

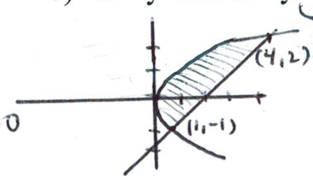
   points $(0, -2)$, $(2, 0)$

$2 + y = y^2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$



$$\int_{-1}^2 [(2+y) - y^2]\,dy$$

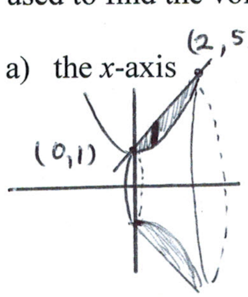
3. For the region bounded by $y = x^2 + 1$ and $y = 2x + 1$, set up the integral that would be used to find the volume of the solid produced by rotating the region about:

   a) the *x*-axis

$x^2 + 1 = 2x + 1$

$x^2 = 2x$

$x = 2, 0$



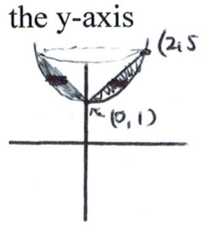
washer, dx

$R(x) = 2x + 1$

$r(x) = x^2 + 1$

$$V = \pi \int_0^2 [(2x+1)^2 - (x^2+1)^2]\,dx$$

   b) the y-axis



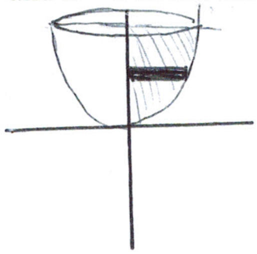
washer, dy

$R(y) = \sqrt{y-1}$

$r(y) = \frac{y-1}{2}$

$$V = \pi \int_1^5 \left[(y-1) - \frac{(y-1)^2}{4}\right] dy$$

4. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.



disk, dy

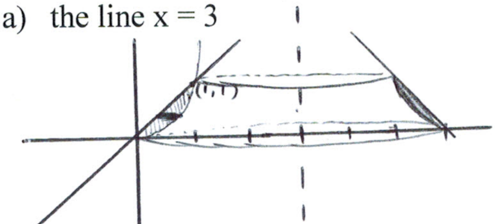
$R(y) = y^{1/3}$

$V = \pi \int_0^8 y^{2/3}\, dy$

$19.2\pi = \frac{96}{5}\pi$

5. For the region enclosed by $y = x$ and $y = x^2$, set up the integral that would be used to find the volume of the solid obtained by rotating the region about:

a) the line $x = 3$



washer

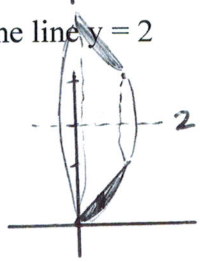
dy

$R(y) = 3 - y$

$r(y) = 3 - \sqrt{y}$

$V = \pi \int_0^1 \left[(3-y)^2 - (3-\sqrt{y})^2\right] dy$

b) the line $y = 2$



washer, dx

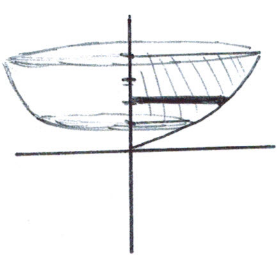
$R(x) = 2 - x^2$

$r(x) = 2 - x$

$V = \int_0^1 \left[(2-x^2)^2 - (2-x)^2\right] dx$

$y = \frac{1}{4}x^2$

6. Find the volume of the solid generated by revolving the area bounded by $x^2 = 4y$, the y-axis, and the lines $y = 1$ as well as $y = 4$ about the y-axis.



disk, dy

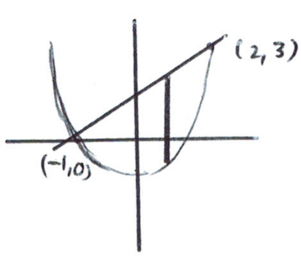
$R(y) = \sqrt{4y} = 2\sqrt{y}$

$V = \pi \int_1^4 4y\, dy$

$= 30\pi$

wording, format

7. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated square cross section taken perpendicular to the x-axis.



$x + 1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

8.1

$s(x) = \text{top} - \text{bottom} = x + 1 - (x^2 - 1) = -x^2 + x + 2$

$V = \int_{-1}^{2} [s(x)]^2 dx = \int_{-1}^{2} (-x^2 + x + 2)^2 dx$

Set up the integral to find the volume of the solid described by each situation. You do NOT need to evaluate any of these.

7. The base of the volume is a circle centered at the origin with radius 5. The cross sections perpendicular to the x-axis are:

$x^2+y^2=25$

$y=\pm\sqrt{25-x^2}$

a) Squares

$V=\int [s(x)]^2\,dx$

$s(x)=2\sqrt{25-x^2}$

$V=\int_{-5}^{5} 4(25-x^2)\,dx = 4\int_{-5}^{5}(25-x^2)\,dx$

$A=s^2$

b) Equilateral triangles

$A=\frac{\sqrt{3}\,s^2}{4}$

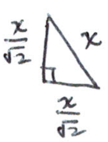
$V=\frac{\sqrt{3}}{4}\int_{-5}^{5} 4(25-x^2)\,dx = \sqrt{3}\int_{-5}^{5} 25-x^2\,dx$

c) Isosceles right triangles with leg on the base

$A=\frac{1}{2}s^2$

$V=2\int_{-5}^{5}(25-x^2)\,dx$

d) Isosceles right triangles with hypotenuse on the base



$A=\frac{1}{2}\left(\frac{x}{\sqrt{2}}\right)\left(\frac{x}{\sqrt{2}}\right)=\frac{x^2}{4}$

$V=\frac{1}{4}\int_{-5}^{5} 4(25-x^2)\,dx = \int_{-5}^{5}(25-x^2)\,dx$

e) Semi-circles

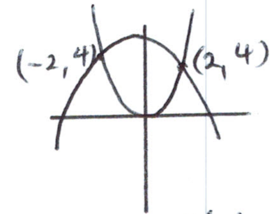
$A=\frac{\pi r^2}{2}$

$V=\frac{\pi}{2}\int_{-5}^{5}(25-x^2)\,dx$

$r=\sqrt{25-x^2}$

NEXT PAGE >>>

8. The base of the volume is the region bounded by $y = 8 - x^2$ and $y = x^2$. The cross sections perpendicular to the x-axis are:



$x^2 = 8 - x^2$

$2x^2 = 8$

$x^2 = 4$

$s(x) = \text{top} - \text{bottom}$

$8 - x^2 - x^2$

$s = 8 - 2x^2$

a) Squares

$A = s^2$

$V = \int_{-2}^{2} (8 - 2x^2)^2 \, dx$

b) Equilateral triangles

$A = \frac{\sqrt{3}}{4} s^2$

$V = \frac{\sqrt{3}}{4} \int_{-2}^{2} (8 - 2x^2)^2 \, dx$

c) Isosceles right triangles with leg on the base

$A = \frac{1}{2} s^2$

$V = \frac{1}{2} \int_{-2}^{2} (8 - 2x^2)^2 \, dx$

d) Isosceles right triangles with hypotenuse on the base

$A = \frac{s^2}{4}$

$V = \frac{1}{4} \int_{-2}^{2} (8 - 2x^2)^2 \, dx$

e) Semi-circles

$A = \frac{\pi r^2}{2}$

$r = 4 - x^2$

$V = \int_{-2}^{2} \frac{\pi (4 - x^2)^2}{2} \, dx = \frac{\pi}{2} \int_{-2}^{2} (4 - x^2)^2 \, dx$