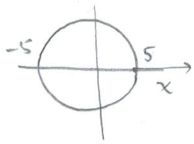
Name: Key

Calculus: Solids with Known Cross-Sections

1. Find the volume of the solid with circular base of diameter 10 cm and whose cross-sections perpendicular to a given diameter are equilateral triangles.



$A_{\triangle} = \frac{\sqrt{3}}{4}s^2$ $\quad s = 2\sqrt{25-x^2}$

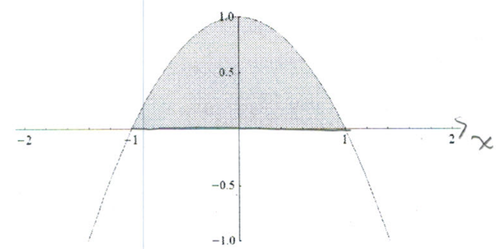
$x^2+y^2=25$

$y=\sqrt{25-x^2}$

$2\frac{\sqrt{3}}{4}\int_0^5 \left(2\sqrt{25-x^2}\right)^2 dx = 2\sqrt{3}\int_0^5 (25-x^2)\,dx$

$\frac{500\sqrt{3}}{3} \approx 288.68$

2. The base of a solid is the region bounded by the graph of $y=1-x^2$ and the x-axis. For this solid, each cross section perpendicular to the x-axis is a rectangle with height three times the base. What is the volume of this solid?

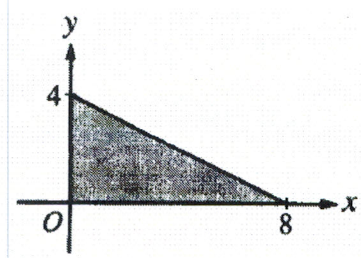


$A_{\square} = b\cdot h = b\cdot 3b = 3b^2$

$b = 1-x^2$

$2\int_0^1 3(1-x^2)^2 dx = 6\int_0^1 (1-x^2)^2 dx = 3.2$

3. The base of a solid is the region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?



$y = -\frac{1}{2}x+4$

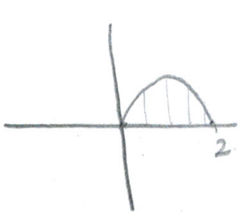
A. 12.566 B. 14.661 C. 16.755
D. 67.021 E. 134.041

$A = \frac{\pi r^2}{2}$ $\quad r(x) = (-\frac{1}{2}x+4)/2 = -\frac{x}{4}+2$

$\frac{\pi}{2}\int_0^8 \left(-\frac{x}{4}+2\right)^2 = \frac{16\pi}{3}$

4. The region bounded by the graph of $y=2x-x^2$ $= x(2-x)$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

A. 1.333 B. 1.067 C. 0.577 D. 0.462 E. 0.267



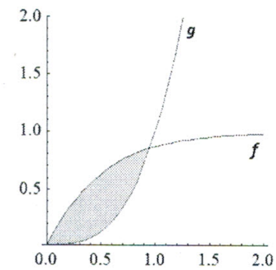
$s = -x^2+2x$

$\frac{\sqrt{3}}{4}\int_0^2 (-x^2+2x)^2 dx = \frac{4\sqrt{3}}{15}$

$1 - e^{-x} = x^3$

$x \approx .825$

5. The region in Quadrant I bounded by the graph of $f(x) = 1 - e^{-x}$ and $g(x) = x^3$ is the base of a solid. Find the volume of this solid, if



(a) For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid.

$A_\Delta = \frac{1}{2}bh = \frac{1}{2}b^2 \qquad b = (1 - e^{-x}) - x^3$

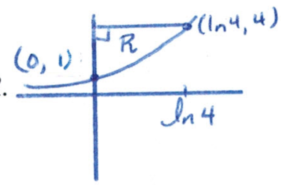
$\frac{1}{2}\int_0^{.825} (1 - e^{-x} - x^3)^2\,dx \approx .01592$

(b) For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with the hypotenuse across the base of the solid.

$\frac{1}{2}\int_0^{.825} \left(\frac{1 - e^{-x} - x^3}{\sqrt{2}}\right)^2 = .00796$

6. Let R be the region in Quadrant I bounded by the graph of $y = e^x$, the y-axis, and the horizontal line $y = 4$.

(a) Find the area of R.



$\int_0^{\ln 4} (4 - e^x)\,dx$

≈ 2.545

$4 = e^x$

$\ln 4 = x \ln e$

$x = \ln 4$

(b) The region R is the base of a solid. For this solid, each cross section **perpendicular to the *y*-axis** is a square. Find the volume of this solid.

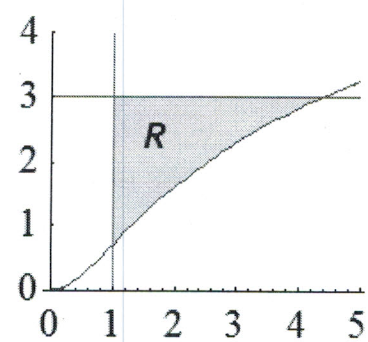
$s(y) = (\ln y)^2$

$y = e^x$

$x = \ln y$

$\int_1^4 (\ln y)^2\,dy \approx 2.597$

7. Let R be the region bounded by the graph of $y = \ln(x^2 + 1)$, the horizontal line $y = 3$, and the vertical line $x = 1$, as shown in the figure.



(a) Find the area of R.

$\ln(x^2+1) = 3$

$x^2 = e^3 - 1$

$x = \sqrt{e^3 - 1}$

$A = \int_1^{\sqrt{e^3-1}} [3 - \ln(x^2+1)]\,dx$

≈ 3.310

(b) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a triangle with height equal to twice the length of the base. Find the volume of this solid.

$A = \frac{1}{2}bh = \frac{1}{2}b(2b) = b^2$

$\int_1^{\sqrt{e^3-1}} [3 - \ln(x^2+1)]^2\,dx \approx 4.722$